International Journal of Theoretical Physics, Vol. 41, No. 7, July 2002 (© 2002)

Models for a Finite Universe[1]

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Received December 6, 2001

A method of constructing 3-dimensional hyperbolic manifolds is described. Because of their high degree of symmetry, these may be suitable models for a finite universe. Because their group of symmetries is different from any in the list of manifolds given in Hodgeson and Weeks (available at ftp://ftp.northnet.org/pub/weeks/snappea/closedcensus), it is claimed that these are new.

KEY WORDS: 3-dimensional hyperbolic manifold; model of the Universe.

In a paper in *Scientific American* Luminet, Starkman, and Weeks (1999) ask, "Is space finite?" And it is generally accepted that the local geometry of space should be hyperbolic. Yet no examples of finite 3-dimensional hyperbolic spaces with much symmetry are known. The examples given in the web site ftp://ftp.northnet.org/pub/weeks/snappea/closedcensus have very restricted groups of Automorphisms, none, in fact, approaching 60 which is the order of $\boldsymbol{A}_5$ which is, somehow, inextricably contained in the groups of the manifolds presented here. This seems to be, because the ones considered there appear to be constructed by identifying opposite faces of a polyhedron. The ones here are of a completely different type. Their Automorphism Group acts transitively on the points and at each point acts just as does the full Automorphism Group of Hyperbolic 3-space, which is the same as the action at a point of the Automorphism Group of Euclidean 3-space, i.e., as the Orthogonal Group $\boldsymbol{O}_3$.

The construction begins with 3-dimensional hyperbolic space, $\boldsymbol{H}^3$. It is well known that this space can be tessellated by a special type of dodecahedron: Each of the 12 pentagons on its surface has each of its angles a right angle. At each vertex, 6 edges, 12 faces, and 8 dodecahedra come together, each edge has two end points, and is incident with four faces and four dodecahedra, each face has five vertices, five edges, and is the face of two dodecahedra. At each vertex, the

[1] All calculations reported in the paper were carried out using John Cannon's Algebraic Computations package MAGMA (Bosma *et al.*, 1997).

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0020-7748/02/0700-1201/0 © 2002 Plenum Publishing Corporation

Geometry is, locally, the same as the tessellation of 3-dimensional space by cubes, but the curvature of the space allows the solids to be dodecahedra.

The group of this tessellation is described in terms of flags.

Definition. An incident (point, edge, face, solid) four-tuple of the tessellation is called a flag.

Definition. Let (v, e, f, s) be one of the flags and $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ be the unique reflections of $\boldsymbol{H}^3$ defined by the requirements:

$\boldsymbol{a}$ fixes v, e, f and moves s onto the other dodecahedron having f as a face.
$\boldsymbol{b}$ fixes v, e, s and moves f onto the other face of s having e as an edge.
$\boldsymbol{c}$ fixes v, f, s and moves e onto the other edge of f having v as vertex.
$\boldsymbol{d}$ fixes e, f, s and moves v onto the other vertex of e.

It is not hard to see that $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ satisfy the relations:

$$\boldsymbol{a}^2 = \boldsymbol{b}^2 = \boldsymbol{c}^2 = \boldsymbol{d}^2 = (\boldsymbol{ab})^4 = (\boldsymbol{bc})^3 = (\boldsymbol{cd})^5 = (\boldsymbol{ac})^2 = (\boldsymbol{ad})^2 = (\boldsymbol{bd})^2 = 1.$$

This group, which is called the Coxeter Group [**4**,**3**,**5**] is, in fact, a complete description of the group of the tessellation.

Now, let [4,3,5] be a Group generated by a,b,c,d which satisfies, among others, these relations, but which is such that the extra relations do not destroy any of the original ones: i.e., in the group, all these relations are satisfied faithfully: i.e., a has order 2, ... ab has order 4, and so on. Then the quotient structure $\boldsymbol{Q}$ of $\boldsymbol{H}^3$ is a manifold which can be constructed as follows:

For each right coset of $\langle b,c,d\rangle$ in [4,3,5], $\boldsymbol{Q}$ is to have one dodecahedron of the type of the tessellation.

For each right coset of $\langle a,c,d\rangle$ in [4,3,5], $\boldsymbol{Q}$ is to have one pentagon of the type of the tessellation and $\langle a,c,d\rangle g$ is incident with the dodecahedron $\langle b,c,d\rangle h$ if and only if

$$\langle a,c,d\rangle gh^{-1}\langle b,c,d\rangle = \langle a,c,d\rangle\langle b,c,d\rangle.$$

For each right coset of $\langle a,b,d\rangle$ in [4,3,5], $\boldsymbol{Q}$ is to have an edge of the type of the tessellation and $\langle a,b,d\rangle g$ is incident with the face $\langle a,c,d\rangle h$ if and only if

$$\langle a,b,d\rangle gh^{-1}\langle a,c,d\rangle = \langle a,b,d\rangle\langle a,c,d\rangle.$$

For each right coset of $\langle a,b,c\rangle$ in [4,3,5], $\boldsymbol{Q}$ is to have a vertex of the type of the tessellation and $\langle a,b,c\rangle g$ is incident with the edge $\langle a,b,d\rangle h$ if and only if

$$\langle a,b,c\rangle gh^{-1}\langle a,b,d\rangle = \langle a,b,c\rangle\langle a,b,d\rangle.$$

In this way the structure of a manifold is built up. To see this, notice, for example that the dodecahedron $\langle b,c,d\rangle g$ has, as its faces, all those of the form $\langle a,c,d\rangle h$ such that

$$\langle b,c,d\rangle gh^{-1}\langle a,c,d\rangle = \langle b,c,d\rangle\langle a,c,d\rangle$$

There is thus one for each right coset of $\langle c,d\rangle$ in $\langle b,c,d\rangle$, i.e., one for each face of the dodecahedron fixed by $\langle b,c,d\rangle$.

This describes a method of constructing a manifold consisting of a finite number of dodecahedra from the finite group [4,3,5].

Alternatively, the manifold can be described as the quotient structure induced on $\boldsymbol{H}^3$ by the homomorphism [**4**,**3**,**5**] $\to$ [4,3,5] defined by $\boldsymbol{a} \to a$, $\boldsymbol{b} \to b$, $\boldsymbol{c} \to c$, $\boldsymbol{d} \to d$.

The group [4,3,5] acts on the tessellation by right multiplication: for example, the edge $\langle a,b,d\rangle hg$ is adjacent to the face $\langle a,c,d\rangle kg$ if and only if $\langle a,b,d\rangle hg(kg)^{-1}$ $\langle a,c,d\rangle = \langle a,b,d\rangle\langle a,c,d\rangle$ if and only if $\langle a,b,d\rangle hk^{-1}\langle a,c,d\rangle = \langle a,b,d\rangle$ if and only if $\langle a,b,d\rangle h$ is adjacent to $\langle a,c,d\rangle k$.

There is one obvious method of obtaining such a finite group: add suitable relations to the ones for [**4**,**3**,**5**]. For example you might try adding $(\boldsymbol{abcd})^n = \boldsymbol{1}$ for some n. This does not give a satisfactory answer for any value of n. Values up to 10 give groups which do not represent the relations exactly, and values of n above 10 give groups which are either so *big* that MAGMA cannot handle them or are infinite.

However, there is a more fruitful method. The Low Index Subgroups Algorithm will give you all the subgroups of a finitely presented group $\boldsymbol{G}$ of a given index. If you have a subgroup $\boldsymbol{H}$ of index n, then the representation of $\boldsymbol{G}$ on the right cosets of $\boldsymbol{H}$ will give you a subgroup, G, of the Symmetric Group of index n, which satisfies the relations of $\boldsymbol{G}$ and others. If the others are not too restrictive, the relations in the old group will be faithfully represented in the new group.

Applying the algorithm in MAGMA to $\boldsymbol{G}$ at index 120 leads to 45,991 potential manifolds. Of course, any number will do in place of 120, but 120 is a particularly fruitful number: Those groups in which the subgroup $\boldsymbol{H}$ is complementary to the stabilizer of one of the dodecahedra of the original tessellation give rise to an orbifold structure which was the subject of Lorimer (1995).

Rather than give details for a bewildering number of groups, I set out to find one of each order. The others of that order may or may not be isomorphic to the chosen one. At least, the spaces from different orders have different volume and are not isometric.

However, the procedure when carried out on a Sun Enterprise 4000, crashed after 24 days when fewer than 9000 of the groups had been processed. The details of the resulting 201 groups are given, probably enough for present requirements. The table gives the order, its prime factorization, and a composition series for the group. A copy of the MAGMA program and a limited amount of the output can be viewed on the author's web site: http:www.math.auckland.ac.nz/∼lorimer. And, finally, in this introduction, some facts about the manifolds and their groups are as follows:

(1) Although the manifold is made up of dodecahedra, they are in no way present in the final structure. They disappear in the uniformity of the construction, i.e., each point in it has as a neighbourhood an open set of

$\boldsymbol{H}^3$, a vertex has eight octants, an edge has four quadrants, and a face has two hemispheres incident with it. They have no physical presence, and it makes no sense to look for them.

(2) The group G acts as a group of the manifold by right multiplication and has order 120 $\times$ number of dodecahedra in the manifold.

(3) The volume of the manifold is equal to the volume of one of the dodecahedra times the order of the group divided by 120.

(4) The full group of the manifold acts transitively on its points and the stabilizer of each point acts on it in the same way as the stabilizer of a point in $\boldsymbol{H}^3$, which is the same as the action of the symmetry group of 3-D Euclidean space at a point, i.e., it is the orthogonal group $\boldsymbol{O}_3(\mathbf{R})$.

(5) None of these manifolds are contained in the list in Hodgeson and Weeks, because none of them have an Automorphism Group of order even approaching 120 which is the order of A_5, a group which is tied, inextricably, into the groups here.

(6) It is thought that because of their uniformity, that these manifolds might make a better model of the Universe than any in that list.

(7) Each homomorphism of [**4**,**3**,**5**] onto one of the groups [4,3,5] implies the existence of a normal subgroup of **[4**,**3**,**5]** having [4,3,5] as its quotient group. As the intersection of two normal subgroups is also normal, the quotient of the intersection of any two of these normal subgroups in **[4**,**3**,**5]** is also a normal subgroup, and its quotient group is of the same type, but of larger order, thus giving rise to a manifold of greater volume than either of them. And so is the quotient group of the intersection of any three of them, or any four, or for that matter of all of the groups in the table, or even all of the 45,990 groups in the original list, a group of unimaginably huge order leading to a manifold of unimaginably huge volume.

(8) It is worth noting, in passing, that none of the manifolds are simply connected, because any extra relation beyond the basic 10 implies a loop in the space which cannot be reduced by homotopy.

The Groups: Each box contains

The number of the group in the list
Its order
The prime factorization of its order
A composition series of it from G down to 1

Notation:

Z_n is the cyclic group of order n.
A_n is the alternating group of degree n.
PSL(2,q) is the special projective linear group over the field of order q.

Sp(6,2) is the simple group of order 1,451,520, which is the 6-dimensional symplectic group over the field of order 2.

Number of Group: 1

Order: 55099802880

Factored Order: $2^8\ 3^{16}\ 5$

Composition Series: $A_5 3Z_2 3Z_3 2Z_2 3Z_3 Z_2 9Z_3$

Number of Group: 2

Order: 3840

Factored Order: $2^7\ 5\ 7$

Composition Series $A_5 6Z_2$

Number of Group: 3

Order: 1920

Factored Order: $2^6\ 3\ 5$

Composition Series $A_5 6Z_2$

Number of Group: 4

Order: 171038672220036805322342400

Factored Order: $2^{64}\ 3^{2}\ 5^{2}\ 7^{2}\ 11^{2}$

Composition Series: $2Z_2 2(PSL(2,29))29Z_2$

Number of Group: 5

Order: 464380231680000000000

Factored Order: $2^{29}\ 3^{11}\ 5^{11}$

Composition Series: $2(A_5Z_2)4(Z_2A_5)A_5Z_2 4A_5$

Number of Group: 6

Order: 7680

Factored Order: $2^{9}\ 3\ 5$

Composition Series: $A_5 7Z_2$

Number of Group: 7

Order: 464380231680000000000

Factored Order: $2^{29}\ 3^{11}\ 5^{11}$

Composition Series: $2(A_5Z_2)4(Z_2A_5)A_5Z_24A_5$

Number of Group: 8

Order: 274005602831007697894197166080000000000000

Factored Order: $2^{51}3^{25}5^{13}7^{6}$

Composition Series: $A_52Z_26(Z_2A_{10})$

Number of Group: 9

Order: 453497320

Factored Order: $2^{9}\ 3^{11}\ 5$

Composition Series: $A_5Z_2Z_3Z_24(Z_3Z_2)2Z_3Z_23Z_3$

Number of Group: 10

Order: 470561093629261335377215488000000000000000

Factored Order: $2^{81}\ 3^{13}\ 5^{13}$

Composition Series: $A_5 2 Z_2 5 (A_5 4 Z_2 A_5 Z_2) 2 (A_5 4 Z_2)$

Number of Group: 11

Order: 16717688340480000000000000

Factored Order: $2^{33}\ 3^{13}\ 5^{13}$

Composition Series: $A_5 2 Z_2 A_5 5 (A_5 Z_2 A_5) A_5$

Number of Group: 12

Order: 36390768678310017761280000000000000

Factored Order: $2^{45}\ 3^{25} 5^{13}$

Composition Series: $A_5 2 Z_2 A_6 5 (A_6 Z_2 A_6) A_6$

Number of Group: 13

Order: 245760

Factored Order: 2^{14} 3 5

Composition Series: $A_5 12 Z_2$

Number of Group: 14

Order: 14511882240000000000

Factored Order: 2^{2} $3^{11} 5^{11}$

Composition Series: $2(A_5 Z_2) 9 A_5$

Number of Group: 15

Order: 4281337544234495279596830720000000000000

Factored Order: 2^{45} 3^{25} 5^{13} 7^{6}

Composition Series: $A_5 Z_2 6 A_{10}$

Number of Group: 16

Order: 120000000

Factored Order: $2^9\ 3\ 5^7$

Composition Series: $A_5Z_2Z_52Z_25(Z_5Z_2)$

Number of Group: 17

Order: 2807929681968365568000000000000

Factored Order: $2^{39}3^{21}5^{11}$

Composition Series: $A_5Z_2A_62Z_24(A_6Z_2)5(A_5)$

Number of Group: 18

Order: 179159040000000

Factored Order: $2^{20}\ 3^7\ 5^7$

Composition Series: $2(A_5Z_2)4(Z_2A_5)A_5$

Number of Group: 19

Order: 358318080000000

Factored Order: 2^{21} 3^7 5^7

Composition Series: $2(A_5Z_2)5(Z_2A_5)$

Number of Group: 20

Order: 300578991243264000000

Factored Order: 2^{44} 3^7 5^7

Composition Series: $A_5Z_25(A_55Z_2A_54Z_2)2(A_54Z_2)$

Number of Group: 21

Order: 601157982486528000000

Factored Order: 2^{45} 3^7 5^7

Composition Series: $A_5Z_2A_56Z_23(A_55Z_2)2(A_54Z_2)$

Number of Group: 22

Order: 235280546814630667688607744000000000000000

Factored Order: 2^{80} 3^{13} 5^{13}

Composition Series: $A_5Z_25(A_55Z_2A_54Z_2)2(A_54Z_2)$

Number of Group: 23

Order: 601157982486528000000

Factored Order: 2^{45} 3^{7} 5^{7}

Composition Series: $A_5Z_2(A_56Z_2)4(A_55Z_2)A_54Z_2$

Number of Group: 24

Order: 16717688340480000000

Factored Order: 2^{27} 3^{13} 5^{7}

Composition Series: $A_5Z_26(Z_2A_6)$

Number of Group: 25
Order: 235280546814630667688607744000000000000000
Factored Order: $2^{80}3^{13}5^{13}$
Composition Series: $A_5Z_25(A_55Z_2A_54Z_2)2(A_54Z_2)$

Number of Group: 26
Order: 3750000000000000
Factored Order: $2^{10}35^{13}$
Composition Series: $A_54Z_2Z_54(Z_2Z_5)7Z_5$

Number of Group: 27
Order: 1055531162664960000000000000
Factored Order: $2^{58}35^{13}$
Composition Series: $A_53Z_25(Z_35Z_2)7(Z_34Z_2)$

Number of Group: 28

Order: 1370028014155038489470985830400000000000000

Factored Order: $2^{50}3^{25}5^{13}7^{6}$

Composition Series: $A_5Z_2A_{10}2Z_23(A_{10}Z_2)2A_{10}$

Number of Group: 29

Order: 83588441702400000000000000

Factored Order: $2^{32}3^{13}5^{13}$

Composition Series: $A_52Z_22A_54(A_5Z_2A_5)2A_5$

Number of Group: 30

Order: 2013265920000000

Factored Order: $2^{33}35^{7}$

Composition Series: $A_5Z_2Z_37Z_23(Z_35Z_2)2(Z_34Z_2)$

Number of Group: 31

Order: 334353766809600000000000

Factored Order: $2^{34}3^{13}5^{13}$

Composition Series: $A_5 3 Z_2 5 (A_5 Z_2) 7 A_5$

Number of Group: 32

Order: 7712572067544714950605700441633459533774848000000

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Factored Order: $2^{99}3^{25}5^{13}7^{6}$

Composition Series: $A_5 Z_2 6 (A_{10} 9 Z_2)$

Number of Group: 33

Order: 941122187258522670754430976000000000000

Factored Order: $2^{82}3^{13}5^{13}$

Composition Series: $A_5 Z_2 A_5 6 Z_2 4 (A_5 5 Z_2) 7 (A_5 4 Z_2)$

Number of Group: 34
Order: 1008575732230069734604800000000
Factored Order: $2^{69}3^{7}5^{7}$
Composition Series: $A_5Z_25(A_59Z_2)A_58Z_2$

Number of Group: 35
Order: 504287866115034867302400000000
Factored Order: $2^{68}3^{7}5^{7}$
Composition Series: $A_52Z_24(A_59Z_2)2(A_58Z_2)$

Number of Group: 36
Order: 18750000000000000
Factored Order: $2^{13}3\,5^{13}$
Composition Series: $A_53Z_2Z_54(Z_5Z_2Z_5)3Z_5$

Number of Group: 37

Order: 527765581332480000000000000

Factored Order: $2^{57}35^{13}$

Composition Series: $A_53Z_24(Z_54Z_2Z_55Z_2)4(Z_54Z_2)$

Number of Group: 38

Order: 2612138803200000000

Factored Order: $2^{21}3^{13}5^{7}$

Composition Series: $A_5Z_26A_5$

Number of Group: 39

Order: 5224277606400000000

Factored Order: $2^{22}3^{13}5^{7}$

Composition Series: $A_5Z_2A_6Z_25A_6$

Number of Group: 40

Order: 26121388032000000000000

Factored Order: $2^{27}3^{13}5^{13}$

Composition Series: $A_5Z_2 12A_5$

Number of Group: 41

Order: 18786186952704000000

Factored Order: $2^{40}3^{7}5^{7}$

Composition Series: $A_5Z_2A_5Z_2 5(A_5 4Z_2)$

Number of Group: 42

Order: 73521708795720836526899200000000000

Factored Order: $2^{16}3^{7}5^{7}$

Composition Series: $2(A_5Z_2)5A_5$

Number of Group: 43
Order: 11197440000000
Factored Order: $2^{16}3^{7}5^{7}$
Composition Series: $2(A_5Z_2)5A_5$

Number of Group: 44
Order: 1006329600000000000000
Factored Order: $2^{75}3^{13}5^{13}$
Composition Series: $A_5Z_2 12(A_5 4Z_2)$

Number of Group: 45
Order: 1197440000000
Factored Order: $2^{16}3^{7}5^{7}$
Composition Series: $2(A_5Z_2)5A_5$

Number of Group: 46

Order: 559872000000

Factored Order: $2^{15}3^{7}5^{7}$

Composition Series: $A_5Z_26A_5$

Number of Group: 47

Order: 261213880320000000000000

Factored Order: $2^{27}3^{13}5^{13}$

Composition Series: $A_5Z_212A_5$

Number of Group: 48

Order: 60000000

Factored Order: $2^{8}35^{7}$

Composition Series: $A_52Z_26Z_54Z_2$

Number of Group: 49

Order: 11197440000000

Factored Order: $2^{16}3^{7}5^{7}$

Composition Series: $2(A_5Z_2)5A_5$

Number of Group: 50

Order: 5598720000000

Factored Order: $2^{15}3^{7}5^{7}$

Composition Series: $A_5Z_26A_5$

Number of Group: 51

Order: 100632960000000

Factored Order: $2^{32}35^{7}$

Composition Series: $A_5Z_2Z_56Z_23(Z_55Z_2)2(Z_54Z_2)$

Number of Group: 52

Order: 60000000

Factored Order: $2^8 3 5^7$

Composition Series: $A_5 Z_2 Z_5 2 Z_2 3 (Z_5 Z_2) 2 Z_5$

Number of Group: 53

Order: 522427760640000000000000

Factored Order: $2^{28} 3^{13} 5^{13}$

Composition Series: $A_5 2 Z_2 12 A_5$

Number of Group: 54

Order: 856267508846899055919366144000000000000000

Factored Order: $2^{46} 3^{25} 5^{13} 7^6$

Composition Series: $A_5 Z_2 A_{10} Z_2 5 A_{10}$

Number of Group: 55

Order: 593409600

Factored Order: $2^{6}3^{2}5^{2}7^{2}29^{2}$

Composition Series: $2Z_2 2PSL(2,29)$

Number of Group: 56

Order: 232190115840000000000

Factored Order: $2^{28}3^{11}5^{11}$

Composition Series: $A_5 2Z_2 4(A_5Z_2)6A_5$

Number of Group: 57

Order: 941122187258522670754430976000000

Factored Order: $2^{76}3^{13}5^{7}$

Composition Series: $A_5 2Z_2 6(A_5 9Z_2)$

Number of Group: 58

Order: 147050341759144167305379840000000000000

Factored Order: $2^{76}3^{13}5^{13}$

Composition Series: $A_5 2 Z_2 12(A_5 4 Z_2)$

Number of Group: 59

Order: 15425144135089429901211400883266919067549696000000

0000000

Factored Order: $2^{113}3^{25}5^{13}7^{6}$

Composition Series: $A_5 Z_2 A_{10} 10 Z_2 5(A_{10} 9 Z_2)$

Number of Group: 60

Order: 12023159649730560000000

Factored Order: $2^{46}3^{7}5^{7}$

Composition Series: $A_5 Z_2 A_5 8 Z_2 A_5 6 Z_2 A_5 5 Z_2 3(A_5 4 Z_2)$

Number of Group: 61
Order: 71636160000000
Factored Order: $2^{22}3^{7}5^{7}$
Composition Series: $A_5Z_2A_54Z_2A_52Z_2A_5Z_23A_5$

Number of Group: 62
Order: 2017151464460139469209600000000
Factored Order: $2^{70}3^{7}5^{7}$
Composition Series: $A_5Z_2A_512Z_2A_510Z_2A_59Z_23(A_58Z_2)$

Number of Group: 63
Order: 40265318400000000
Factored Order: $2^{34}35^{7}$
Composition Series: $A_5Z_2Z_58Z_2Z_56Z_2Z_55Z_23(Z_54Z_2)$

Number of Group: 64

Order: 240000000

Factored Order: $2^{10}35^{7}$

Composition Series: $A_5Z_2Z_54Z_2Z_52Z_2Z_5Z_23Z_5$

Number of Group: 65

Order: 226748160

Factored Order: $2^{8}3^{11}5$

Composition Series: $A_5Z_2Z_32Z_22(Z_3Z_2)3Z_3Z_24Z_3$

Number of Group: 66

Order: 83209871127413901442763411832233643807541726063612

45952449277694096000000000000000

Factored Order: $2^{56}3^{28}57^{4}11^{9}13^{5}17^{4}19^{3}23^{3}29^{2}$.31 37 41 43 47 53 59

Composition Series: Z_2A_{60}

Number of Group: 67

Order: 14039648409841827840000000000

Factored Order: $2^{38}3^{21}5^{11}$

Composition Series: $A_5Z_2A_62Z_23(A_6Z_2)6A_6$

Number of Group: 68

Order: 906992640

Factored Order: $2^{10}3^{11}5$

Composition Series: $A_5Z_2Z_34Z_2Z_32Z_2Z_37Z_3$

Number of Group: 69

Order: 62914560

Factored Order: $2^{22}3\ 5$

Composition Series: A_520Z_2

Number of Group: 70

Order: 104544000

Factored Order: $2^8 3^3 5^3 11^2$

Composition Series: $A_5 2 Z_2 2 PSL(2,11)$

Number of Group: 71

Order: 42156302400

Factored Order: $2^6 3^2 5^2$ 29 59

Composition Series: $2 Z_2 PSL(2,59)$

Number of Group: 72

Order: 11197440000000

Factored Order: $2^{16} 3^7 5^7$

Composition Series: $2(A_5 Z_2) 5 A_5$

Number of Group: 73

Order: 5598720000000

Factored Order: $2^{15}3^{7}5^{7}$

Composition Series: $A_5Z_26A_5$

Number of Group: 74

Order: 1006632960000000

Factored Order: $2^{32}35^{7}$

Composition Series: $A_5Z_2Z_56Z_23(Z_55Z_2)2(Z_54Z_2)$

Number of Group: 75

Order: 60000000

Factored Order: $2^{8}3\ 5^{7}$

Composition Series: $A_5Z_2Z_52Z_23(Z_5Z_2)2Z_5$

Number of Group: 76

Order: 5224277606400000000000

Factored Order: $2^{28}3^{13}5^{13}$

Composition Series: $A_5 2Z_2 12A_5$

Number of Group: 77

Order: 85626750884689905591936614400000000000000

Factored Order: $2^{46}3^{25}5^{13}7^{6}$

Composition Series: $A_5 Z_2 A_{10} Z_2 5A_{10}$

Number of Group: 78

Order: 593409600

Factored Order: $2^{6}3^{2}5^{2}7^{2}29^{2}$

Composition Series: $2Z_2 2PSL(2,29)$

Number of Group: 79
Order: 232190115840000000000
Factored Order: $2^{28}3^{11}5^{11}$
Composition Series: $A_5 2 Z_2 4(A_5 Z_2) 6 A_5$

Number of Group: 80
Order: 94112218725852267075443097600000000
Factored Order: $2^{76}3^{13}5^{7}$
Composition Series: $A_5 2 Z_2 6(A_5 9 Z_2)$

Number of Group: 81
Order: 1470503417591441673053798400000000000000
Factored Order: $2^{76}3^{13}5^{13}$
Composition Series: $A_5 2 Z_2 12(A_5 4 Z_2)$

Number of Group: 82

Order: 1542514413508942990121140088326691906754969600000000000000

Factored Order: $2^{100}3^{25}5^{13}7^{6}$

Composition Series: $A_5Z_2A_{10}10Z_25(A_{10}9Z_2)$

Number of Group: 83

Order: 1202315964973056000000

Factored Order: $2^{46}3^{7}5^{7}$

Composition Series: $A_5Z_2A_58Z_2A_56Z_2A_55Z_23(A_54Z_2)$

Number of Group: 84

Order: 716636160000000

Factored Order: $2^{22}3^{7}5^{7}$

Composition Series: $A_5Z_2A_54Z_2A_52Z_2A_5Z_23A_5$

Number of Group: 85

Order: 201715146446013946920960000000

Factored Order: $2^{70}3^{7}5^{7}$

Composition Series: $A_5Z_2A_5 12Z_2A_5 10Z_2A_5 9Z_2 3(A_5 8Z_2)$

Number of Group: 86

Order: 4026531840000000

Factored Order: $2^{34}35^{7}$

Composition Series: $A_5Z_2Z_5 8Z_2Z_5 6Z_2Z_5 5Z_2 3(Z_5 4Z_2)$

Number of Group: 87

Order: 240000000

Factored Order: $2^{10}35^{7}$

Composition Series: $A_5Z_2Z_5 4Z_2Z_5 2Z_2Z_5Z_2 3Z_5$

Number of Group: 88

Order: 226748160

Factored Order: $2^{8}3^{11}5$

Composition Series: $A_5Z_2Z_32Z_22(Z_3Z_2)3Z_3Z_24Z_3$

Number of Group: 89

Order: 832098711274139014427634118322336438075417260636612

4595244927769640960000000000000

Factored Order: $2^{56}3^{28}5^{13}7^{4}11^{9}13^{5}17^{4}19^{3}23^{3}29^{2}31^{2}37\ 41\ 43\ 47\ 53\ 59$

Composition Series: Z_2A_{60}

Number of Group: 90

Order of Group: 418278400000000000

Factored Order: $2^{38}3^{21}5^{11}$

Composition Series: $A_5Z_2A_62Z_23(A_6Z_2)6A_6$

Number of Group: 91

Order: 906992640

Factored Order: $2^{10}3^{11}5$

Composition Series: $A_5Z_2Z_34Z_2Z_32Z_2Z_3Z_27Z_3$

Number of Group: 92

Order: 62914560

Factored Order: $2^{22}35$

Composition Series: A_520Z_2

Number of Group: 93

Order: 5272000

Factored Order: $2^83^35^311^2$

Composition Series: $A_52Z_22PSL(2,11)$

Number of Group: 94
Order: 42156302400
Factored Order: $2^6 3^2 5^2 29^2 59^2$
Composition Series: $2Z_2 2PSL(2,59)$

Number of Group: 95
Order: 95611771622970586308280320
Factored Order: $2^{19} 3^{41} 5$
Composition Series: $A_5 4Z_3 4(Z_2 Z_3) 4Z_2 3Z_3 2(Z_2 Z_3) 2(2Z_2 Z_3) 2(Z_2 Z_3) 24Z_3$

Number of Group: 96
Order: 46785600
Factored Order: $2^6 3^4 5^2 19^2$
Composition Series: $2Z_2 2PSL(2,19)$

Number of Group: 97

Order: 249312238496812892160

Factored Order: $2^{48}3^{11}5$

Composition Series: $A_5Z_2Z_36Z_23(Z_35Z_2Z_34Z_2)3(Z_34Z_2)$

Number of Group: 98

Order: 491520

Factored Order: $2^{15}3\ 5$

Composition Series: A_513Z_2

Number of Group: 99

Order: 498624476993625784320

Factored Order: $2^{49}3^{11}5$

Composition Series: $A_5Z_2Z_36Z_23(Z_35Z_2)Z_34Z_2Z_35Z_24(Z_34Z_2)$

Number of Group: 100

Order: 73872298498691733054629562248565209747409102354882357384887220295434240000000000000

Factored Order: $2^{114}3^{28}5^{14}7^{9}11^{5}13^{4}17^{3}19^{3}23^{2}29^{2}31\ 37\ 41\ 43\ 47\ 53$

Composition Series: $A_{60}59Z_2$

Number of Group: 101

Order: 316144776734882162832374867637043200000000000

Factored Order: $2^{89}3^{21}5^{11}$

Composition Series: $A_5Z_2A_67Z_22(A_66Z_2)A_65Z_22(A_66Z_2)4(A_65Z_2)$

Number of Group: 102

Order: 2903764480

Factored Order: $2^{15}3^{11}5$

Composition Series: $A_52Z_22(Z_32Z_2)Z_3Z_22(Z_32Z_2Z_3Z_2)3Z_3$

Number of Group: 103
Order: 30115909992272725464141791232000000
Factored Order: $2^{81}3^{14}5^{7}$
Composition Series: $A_5 2Z_2 5(A_6 10Z_2)A_6 9Z_2$

Number of Group: 104
Order: 1742400
Factored Order: $2^{6}3^{2}5^{2}11^{2}$
Composition Series: $2Z_2 2PSL(2,11)$

Number of Group: 105
Order: 1451520
Factored Order: $2^{9}3^{4}5\ 7$
Composition Series: Sp(6,2)

Number of Group: 106

Order: 384741108791377920000000

Factored Order: $2^{51}3^{7}5^{7}$

Composition Series: $A_5Z_2A_57Z_24(A_56Z_2)A_55Z_2$

Number of Group: 107

Order of Group: 22932357120000000

Factored Order: $2^{27}3^{7}5^{7}$

Composition Series: $A_5Z_2A_55Z_2A_56Z_29(A_55Z_2)A_54Z_2$

Number of Group: 108

Order: 835884170240000000

Factored Order: $2^{26}3^{13}5^{7}$

Composition Series: $A_5Z_2A_62Z_23(A_6Z_2)2A_6$

Number of Group: 109

Order: 30115909992272725464141791232000000000000

Factored Order: $2^{87}3^{13}5^{13}$

Composition Series: $A_5Z_2A_55Z_2A_56Z_29(A_55Z_2)A_54Z_2$

Number of Group: 110

Order: 106993205379072000000000000

Factored Order: $2^{39}3^{13}5^{13}$

Composition Series: $3(A_5Z_2)10(Z_2A_5)$

Number of Group: 111

Order: 835884172024000000

Factored Order: $2^{26}3^{13}5^{7}$

Composition Series: $A_5Z_2A_62Z_23(A_66Z_2)2A_6$

Number of Group: 112

Order: 30115909992272725464141791232000000000000

Factored Order: $2^{87}3^{13}5^{13}$

Composition Series: $A_5Z_2A_55Z_2A_56Z_29(A_55Z_2)A_54Z_2$

Number of Group: 113

Order: 1069932053790720000000000000

Factored Order: $2^{39}3^{13}5^{13}$

Composition Series: $3(A_5Z_2)10(Z_2A_5)$

Number of Group: 114

Order: 64548846862724463014707200000000

Factored Order: $2^{75}3^{7}5^{7}$

Composition Series: $A_5Z_2A_511Z_24(A_510Z_2)A_59Z_2$

Number of Group: 115

Order: 4936046123228617568387648282645414101615902720000000000000

Factored Order: $2^{105}3^{25}5^{13}7^{6}$

Composition Series: $A_5Z_2A_{10}11Z_24(A_{10}10Z_2)A_{10}9Z_2$

Number of Group: 116

Order: 24680230616143087841938241411322707050807951360000000000000

Factored Order: $2^{104}3^{25}5^{13}7^{6}$

Composition Series: $A_5Z_25(A_{10}10Z_2)A_{10}9Z_2$

Number of Group: 117

Order: 33776997205278720000000000000

Factored Order: $2^{63}3\,5^{13}$

Composition Series: $A_5 3 Z_2 Z_5 5 Z_2 Z_5 6 Z_2 Z_5 5 Z_2 3 (Z_5 6 Z_2 Z_5 4 Z_2 Z_5) Z_5 5 Z_2$

$2 (Z_5 4 Z_2)$

Number of Group: 118

Order: 128849018889000000

Factored Order: $2^{39}3\,5^{7}$

Composition Series: $A_5 Z_2 Z_5 8 Z_2 3 (Z_5 6 Z_2) 2 (Z_5 5 Z_2)$

Number of Group: 119

Order: 7680000000

Factored Order: $2^{15}3\,5^{7}$

Composition Series: $A_5 Z_2 Z_5 4 Z_2 4 (Z_5 2 Z_2) Z_5$

Number of Group: 120

Order: 1884369711487782972051947520

Factored Order: $2^{55}3^{21}5$

Composition Series: $A_5 2Z_2Z_3 4Z_2 6Z_3 3Z_2 2(Z_3 2Z_2) 4(Z_3 3Z_2) 7(Z_3 2Z_2)$

Number of Group: 121

Order: 1713824268779520

Factored Order: $2^{15}3^{21}5$

Composition Series: $A_5 2Z_2Z_3 2Z_2 7(Z_3Z_2) 3Z_3Z_2 2Z_3Z_2 7Z_3$

Number of Group: 122

Order: 6755399441055744000000

Factored Order: $2^{58}35^{7}$

Composition Series: $A_5Z_2Z_5 12Z_2Z_5 10Z_2Z_5 9Z_2 3(Z_5 8Z_2)$

Number of Group: 123

Order: 216172782113783808000000

Factored Order: $2^{63}35^{7}$

Composition Series: $A_5Z_2Z_5 12Z_2 3(Z_5 10Z_2)2(Z_5 9Z_2)$

Number of Group: 124

Order: 15057954499613636273207089561600000000

Factored Order: $2^{80}3^{13}5^{7}$

Composition Series: $A_5Z_2 5(A_5 10Z_2)A_5 9Z_2$

Number of Group: 125

Order: 38562860337723574753028502208167297668874240000000

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Factored Order: $2^{98}3^{25}5^{13}7^{6}$

Composition Series: $A_5 3(A_{10} 9Z_2)A_{10} 8Z_2Z_3 9Z_2$

Number of Group: 126

Order: 315179916321896792064000000

Factored Order: $2^{64}3^{7}5^{7}$

Composition Series: $A_5Z_2A_59Z_25(A_58Z_2)$

Number of Group: 127

Order: 337769972052787200000000

Factored Order: $2^{57}35^{7}$

Composition Series: $A_5Z_2Z_511Z_23(Z_59Z_2)2(Z_58Z_2)$

Number of Group: 128

Order: 5916296284641165312O

Factored Order: $2^{38}3^{16}5$

Composition Series: $A_5Z_22(Z_34Z_2)2(Z_32Z_2)Z_33Z_210(Z_32Z_2)$

Number of Group: 129

Order: 479672249091799347744596997383466109259124497130419494818204709764714769774440590868480000000000000

Factored Order: $2^{115}3^{28}5^{14}7^{9}11^{5}13^{4}17^{3}19^{3}23^{2}29^{2}31\ 37\ 41\ 43\ 47\ 53\ 59$

Composition Series: $Z_2A_{60}Z_2$

Number of Group: 130

Order: 150579549961363627320708956160000000

Factored Order: $2^{30}3^{13}5^{7}$

Composition Series: $A_5Z_25(A_610Z_2)A_69Z_2$

Number of Group: 131

Order: 38562860337723574753028502208167297668874240000000000000

Factored Order: $2^{98}3^{25}5^{13}7^{6}$

Composition Series: $A_56(A_{10}9Z_2)$

Number of Group: 132

Order: 315179916321897920640000000

Factored Order: $2^{64}3^{7}5^{7}$

Composition Series: $A_5Z_2A_59Z_25(A_58Z_2)$

Number of Group: 133

Order: 337769972052787200000000

Factored Order: $2^{57}35^{7}$

Composition Series: $A_5Z_2Z_511Z_23(Z_59Z_2)2(Z_58Z_2)$

Number of Group: 134

Order: 5916296284641165312O

Factored Order: $2^{38}3^{16}5$

Composition Series: $A_5Z_22(Z_34Z_2)2(Z_3Z_2)Z_33Z_210(Z_32Z_2)$

Number of Group: 135

Order: 47967224909179934774459699738346610925912449713041

94948182047097647147697744405908684800000000000000

Factored Order: $2^{115}3^{28}5^{14}7^{9}11^{5}13^{4}17^{3}19^{3}23^{2}29^{2}31\ 37\ 41\ 43\ 47\ 53\ 59$

Composition Series: $Z_2A_{60}59Z_2$

Number of Group: 136

Order: 16641974225482780288552682366446728761508345212722

49190489855539281920000000000000

Factored Order: $2^{57}3^{28}5^{14}7^{9}11^{5}13^{4}17^{3}19^{3}23^{2}29^{2}31\ 37\ 41\ 43\ 47\ 53\ 59$

Composition Series: $Z_2A_{60}Z_2$

Number of Group: 137

Order: 101199609247161876480

Factored Order: $2^{15}3^{31}5$

Composition Series: $A_5 3Z_2 2Z_3 Z_2 Z_3 2Z_2 3Z_3 2Z_2 3Z_3 Z_2 3Z_3 2Z_2 6Z_3 2Z_2 12Z_3$

Number of Group: 138

Order: 251658240

Factored Order: $2^{24}35$

Composition Series: $A_5 22 Z_2$

Number of Group: 139

Order: 1006632960

Factored Order: $2^{26}3\ 5$

Composition Series: $A_5 24 Z_2$

Number of Group: 140

Order: 50331480

Factored Order: $2^{25}3\ 5$

Composition Series: $A_5 23 Z_2$

Number of Group: 141

Order: 15728640

Factored Order: $2^{20}3\ 5$

Composition Series: $A_5 18 Z_2$

Number of Group: 142

Order: 1944101689411690832601904786128613079397845553 4261

077799368101195235792268079916908544000000000000000000000

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Factored Order: $2^{104}3^{49}5^{25}7^{12}11^{6}13^{6}17^{6}19^{6}$

Composition Series: $A_2 6 A_{20}$

Number of Group: 143

Order: 977592062275832470661992806722682622600562595644571648000000000000000000000000

Factored Order: $2^{93}3^{49}5^{25}7^{8}$

Composition Series: $A_5 2 Z_2 A_{10} 4 (A_{10} Z_2) 3 A_{10}$

Number of Group: 144

Order: 4887960311379162353309964033613413113002812978222858240000000000000000000000000

Factored Order: $2^{92}3^{49}5^{25}7^{12}$

Composition Series: $A_5 2 Z_2 5 (Z_2 2 A_{10}) 2 A_{10}$

Number of Group: 145

Order: 18195384339155008880640000000000000

Factored Order: $2^{44}3^{25}5^{13}$

Composition Series: $A_5 2 Z_2 2 A_6 Z_2 A_6 Z_2 2 A_6 2 (A_6 Z_2) 5 A_6$

Number of Group: 146

Order: 19201620514192128000000

Factored Order: $2^{15}3^{13}5^{7}19^{6}$

Composition Series: $A_5Z_26PSL(2,19)$

Number of Group: 147

Order: 18750000000000000000000000

Factored Order: $2^{21}3\ 5^{25}$

Composition Series: $A_54Z_22Z_52Z_22Z_53Z_2Z_5Z_23Z_53Z_2Z_5Z_22Z_52Z_24Z_5Z_2$

$Z_52Z_28Z_5$

Number of Group: 148

Order: 19201620514192128000000

Factored Order: $2^{15}3^{13}5^{7}19^{6}$

Composition Series: $A_5Z_26PSL(2,19)$

Number of Group: 149

Order: 3888203378823381665203809572257226158795691106852215559873620239047158453615983381708800000000000000000000000

Factored Order: $2^{105}3^{49}5^{25}7^{12}11^{6}13^{6}17^{6}19^{6}$

Composition Series: $A_5Z_26(A_{20})$

Number of Group: 150

Order: 232900919541184113672192000000000000

Factored Order: $2^{51}3^{25}5^{13}$

Composition Series: $A_52Z_2A_6Z_25(A_62Z_2A_6)A_6$

Number of Group: 151

Order: 937500000000000000000000000

Factored Order: $2^{20}3\ 5^{25}$

Composition Series: $A_5 4Z_2 2Z_5 2Z_2 2Z_5 3Z_2 Z_5 Z_2 3Z_5 3Z_2 Z_5 Z_2 3Z_5 2Z_2 4Z_5 2Z_2$

$8Z_5$

Number of Group: 152

Order: 625658919856532781223675396302516878464360061212525

585472000000000000000000000000

Factored Order: $2^{99}3^{49}5^{25}7^{12}$

Composition Series: $A_5 2Z_2 A_{10} Z_2 5(A_{10} 2Z_2 A_{10}) A_{10}$

Number of Group: 153

Order: 72781537356620035522560000000000000

Factored Order: $2^{46}3^{25}5^{13}$

Composition Series: $A_5 3Z_2 A_6 Z_2 2(A_6 2Z_2) 9A_6$

Number of Group: 154

Order: 625658919856532781223675396302516878464360061212525854720000000000000000000000000

Factored Order: $2^{99}3^{49}5^{25}7^{12}$

Composition Series: $A_5 2Z_2 A_{12} Z_2 5(A_{12} 2Z_2 A_{12}) A_{12}$

Number of Group: 155

Order: 72781537356620035522560000000000000

Factored Order: $2^{46}3^{25}5^{13}$

Composition Series: $A_5 3Z_2 A_6 Z_2 2(A_6 2Z_2) 9A_6$

Number of Group: 156

Order: 23290091954118411367219200000000000000000000000000

Factored Order: $2^{63}3^{25}5^{25}$

Composition Series: $A_5 3Z_2 2A_5 Z_2 2A_5 2Z_2 3(4A_5 2Z_2) 4A_5 Z_2 4A_5$

Number of Group: 157

Order: 149056588506357832750202880000000000000000000000

Factored Order: $2^{69}3^{25}5^{25}$

Composition Series: $A_5 3Z_2 A_5 Z_2 A_5 2Z_2 2A_5 3Z_2 2A_5 Z_2 2A_5 3Z_2 2A_5 Z_2 2A_5$

$2Z_2 4A_5 2Z_2 2A_5 Z_2 6A_5$

Number of Group: 158

Order: 745282942531789163751014400000000000000000000000

Factored Order: $2^{68}3^{25}5^{25}$

Composition Series: $A_5 3Z_2 3(A_5 Z_2) A_5 2Z_2 2(A_5 Z_2) 2A_5 2Z_2 6(A_5 Z_2) 10A_5$

Number of Group: 159

Order: 181953843391550088806400000000000000000000000

Factored Order: $2^{56}3^{25}5^{25}$

Composition Series: $A_5 2Z_2 4(4A_5 Z_2) 8A_5$

Number of Group: 160

Order: 7278153735662003552256000000000000000000000000

Factored Order: $2^{58}3^{25}5^{25}$

Composition Series: $A_5 3Z_2 A_5 Z_2 3A_5 2Z_2 4A_5 2Z_2 16A_5$

Number of Group: 161

Order: 305497519461197647081872752100838319562675811138928640000000000000000000000000

Factored Order: $2^{88}3^{49}5^{25}7^{12}$

Composition Series: $A_5 2Z_2 12A_{10}$

Number of Group: 162

Order: 1527487597305988235409363760504191597813379056946

Factored Order: $2^{87}3^{49}5^{25}7^{12}$

Composition Series: $A_5 Z_2 12A_{12}$

Number of Group: 163

Order: 585937500000000000000000

Factored Order: $2^{10}3 5^{19}$

Composition Series: $A_5 3Z_2 2Z_5 Z_2 Z_5 2Z_2 3Z_5 2Z_2 12Z_5$

Number of Group: 164

Order: 1427006658939585920376169897088382706159158558720000000

Factored Order: $2^{105}3^{37}5^{7}$

Composition Series: $A_5 Z_2 2(A_5 Z_2 5(Z_3 3Z_2))3(A_5 Z_2 3(Z_3 3Z_2)Z_3 2Z_2 Z_3 3Z_2))$
$A_5 Z_2 3(Z_3 3Z_2)2(Z_3 2Z_2)$

Number of Group: 165

Order: 36390768678310017761280000000000000000000000000

Factored Order: $2^{57}3^{25}5^{25}$

Composition Series: $A_5 2Z_2 2(2A_5 Z_2)3(4A_5 Z_2)8A_5$

Number of Group: 166

Order: 1137211521197188055040000000000000000000000000

Factored Order: $2^{52}3^{25}5^{25}$

Composition Series: $A_5 2 Z_2 24 A_5$

Number of Group: 167

Order: 113721152119718805504000000000000

Factored Order: $2^{40}3^{13}5^{13}$

Composition Series: $A_5 2 Z_2 12 A_6$

Number of Group: 168

Order: 20816116986658907840814806099558 40

Factored Order: $2^{83}3^{16}5$

Composition Series: $A_5 Z_2 Z_3 7 Z_2 2(Z_3 6 Z_2) Z_3 5 Z_2 Z_3 6 Z_2 10 (Z_3 5 Z_2)$

Number of Group: 169

Order: 63525747639950280275924090880

Factored Order: $2^{68}3^{16}5$

Composition Series: $A_5 2Z_2 Z_3 6Z_2 2(Z_3 5Z_2) Z_3 2Z_2 Z_3 4Z_2 Z_3 2Z_2 Z_3 6Z_2 Z_3 2Z_2$

$Z_3 8Z_2 2(2(Z_3 2Z_2) Z_3 8Z_2))$

Number of Group: 170

Order: 16106127360

Factored Order: $2^{30}35$

Composition Series: $A_5 28Z_2$

Number of Group: 171

Order: 8053063680

Factored Order: $2^{29}35$

Composition Series: $A_5 27Z_2$

Number of Group: 172

Order: 140396484098418278400000000000000000000

Factored Order: $2^{48}3^{21}5^{21}$

Composition Series: $A_5 3Z_2 4A_5 2Z_2 4A_5 Z_2 12A_5$

Number of Group: 173

Order: 51331179029159938820501168550051840000000000000000

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Factored Order: $2^{68}3^{41}5^{21}$

Composition Series: $A_5 Z_2 2(2Z_2 4A_5) Z_2 12A_5$

Number of Group: 174

Order: 47805885811485293154140160

Factored Order: $2^{18}3^{41}5$

Composition Series: $A_5 4Z_2 2(Z_3 Z_2) 2Z_3 Z_2 Z_3 Z_2 3Z_3 3Z_2 2(Z_3 Z_2) 3Z_3 Z_2$

$2(Z_3 Z_2) 25Z_3$

Number of Group: 175

Order: 7019824204920913920000000000000000000000

Factored Order: $2^{47}3^{21}5^{21}$

Composition Series: $A_5 3 Z_2 4 A_5 2 Z_2 16 A_5$

Number of Group: 176

Order: 39985984726747729358052757830893568000000

Factored Order: $2^{34}3^{46}5^{6}7^{5}$

Composition Series: $A_5 Z_2 5(A_8 7 Z_3)$

Number of Group: 177

Order: 34479393552164879285738065274326374762863976415588

44446566125898982576455695473942911078326966681600000000

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Factored Order: $2^{108}3^{51}5^{21}7^{15}11^{10}13^{5}17^{5}19^{5}23^{5}$

Composition Series: $A_5 Z_2 5 A_{24}$

Number of Group: 178

Order: 399607733956902912000000

Factored Order: $2^{33}3^{11}5^{6}7^{5}$

Composition Series: $A_5Z_25A_8$

Number of Group: 179

Order: 3227442343136223150735360

Factored Order: $2^{68}3^{7}5$

Composition Series: $A_52Z_2Z_32Z_2Z_33Z_2Z_39Z_2Z_313Z_2Z_317Z_2Z_320Z_2$

Number of Group: 180

Order: 34479393552164879285738065274326374762863976641558

8444465661258989825764556954739429111078323290000000000

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Factored Order: $2^{108}3^{51}5^{21}7^{15}11^{10}13^{5}17^{5}19^{5}23^{5}$

Composition Series: $A_5Z_25A_{24}$

Number of Group: 181

Order: 399607733956902912000000

Factored Order: $2^{33}3^{11}5^{6}7^{5}$

Composition Series: $A_5Z_25A_8$

Number of Group: 182

Order: 322744234313622315073536O

Factored Order: $2^{68}3^{7}5$

Composition Series: $A_52Z_2Z_32Z_2Z_33Z_2Z_39Z_2Z_313Z_2Z_317Z_2Z_320Z_2$

Number of Group: 183

Order: 33447514567245635287940590270451862933763731665690

14905147835676150816778622481494683437082635992490654078

026062848000000000000000000000000000

Factored Order: $2^{115}3^{58}5^{28}7^{19}11^{10}13^{9}17^{7}19^{6}23^{5}29^{4}31^{3}37^{3}$,$41^{2}43^{2}47^{2}$
$51^{2}59^{2}$61 67 71 73 79 83 89 97 101 103 107 109 113

Composition Series: A_{120}

Number of Group: 184

Order: 69238826530408296215524170450262985254393227649879

65633113248062352404989176669738278588706018806338449479

555334075631636951670473097216000000000000000000000000000

00

Factored Order: $2^{112}3^{56}5^{28}7^{18}11^{10}13^{8}17^{6}19^{6}23^{4}29^{4}31^{2}37^{2}41^{2}43^{2}47^{2}53^{2}$

59^{2}

Composition Series: $2Z_{2}2A_{60}$

Number of Group: 185

Order: 346194132652041481077620852251314926271966138249398281655662403117620249458833486913929435300940316922473977766703781581847583523654860800000000000000000000000000

Factored Order: $2^{111}3^{56}5^{28}7^{18}11^{10}13^{8}17^{6}19^{6}23^{4}29^{4}31^{2}37^{2}41^{2}43^{2}47^{2}53^{2}59^{2}$

Composition Series: $Z_2 2A_{60}$

Number of Group: 186

Order: 16888498602639360000000

Factored Order: $2^{112}3^{56}5^{28}7^{18}11^{10}13^{8}17^{6}19^{6}23^{4}29^{4}31^{2}37^{2}41^{2}43^{2}47^{2}53^{2}59^{2}$

Composition Series: $A_5 Z_2 5(Z_5 9 Z_2) Z_5 8 Z_2$

Number of Group: 187

Order: 64424509440

Factored Order: $2^{32}3\ 5$

Composition Series: $A_5 30 Z_2$

Number of Group: 188

Order: 128849018880

Factored Order: $2^{33}3\ 5$

Composition Series: $A_5 31 Z_2$

Number of Group: 189

Order: 15807238836744108141618743381852160000000000

Factored Order: $2^{8}3^{21}5^{11}$

Composition Series: $A_5 Z_2 4(A_5 6 Z_2) A_5 5 Z_2 A_5 6 Z_2 4(A_5 5 Z_2)$

Number of Group: 190

Order: 2807929681968365568000000000000000000000

Factored Order: $2^{49}3^{21}5^{21}$

Composition Series: $A_5 2 Z_2 5 (A_5 Z_2) 15 A_5$

Number of Group: 191

Order: 66895029134491270575881180540903725867527405525344

08589408121859898481114389650005964960521256960000000000

000000000000000000

Factored Order: $2^{116}3^{58}5^{28}7^{19}11^{10}13^{9}17^{7}19^{6}23^{5}29^{4}31^{3}37^{3}41^{2}43^{2}47^{2}53^{2}$
59^{2} 61 67 71 73 79 83 89 93 97 101 103 107 109 113 117

Composition Series: $Z_2 A_{120}$

Number of Group: 192

Order: 1244225081223482132865219063122312370814621154192 7

089791595584764950907051571146821468160000000000000000000

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Factored Order: $2^{110}3^{49}5^{25}7^{12}11^{6}13^{6}17^{6}19^{6}$

Composition Series: $A_5 6(Z_2 A_{20})$

Number of Group: 193

Order: 65703909157324721690241495744066355200000000000000

Factored Order: $2^{75}3^{41}5^{21}$

Composition Series: $A_5 2Z_2 8(A_6 Z_2)3(A_6 Z_2 A_6)6A_6$

Number of Group: 194

Order: 23845196659183230490198397056205376570660078005018

890766051872604160000000000000000000000

Factored Order: $2^{98}3^{51}5^{21}7^{10}11^{10}$

Composition Series: $A_5Z_23(A_{12}Z_2)2(A_{12}Z_2A_{12})3A_{12}$

Number of Group: 195

Order: 179707499645975396352000000000000000000000

Factored Order: $2^{55}3^{21}5^{21}$

Composition Series: $A_52Z_26(Z_2A_5)A_53(A_5Z_2)A_52(A_5Z_2)7A_5$

Number of Group: 196

Order: 74516239559947595281869990800641801783312743765684

033643912101888000000000000000000000

Factored Order: $2^{93}3^{51}5^{21}7^{10}11^{10}$

Composition Series: $A_5Z_210A_{12}$

Number of Group: 197

Order: 60224136226974557798400000000000

Factored Order: $2^{28}3^{11}5^{11}7^{10}$

Composition Series: $A_5Z_24(PSL(2,11)Z_2)2PSL(2,11)Z_24PSL(2,11)$

Number of Group: 198

Order: 72559411200000000000

Factored Order: $2^{23}3^{11}5^{11}$

Composition Series: $A_5Z_210A_5$

Number of Group: 199

Order: 1026623580583198776410023371001036800000000000000000

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Factored Order: $2^{69}3^{41}5^{21}$

Composition Series: $A_52Z_25(A_5Z_2)15A_5$

Number of Group: 200

Order: 58337602879431427603402823622186762240

Factored Order: $2^{74}3^{31}5$

Composition Series: $A_5 2 Z_2 5 (Z_3 3 Z_2) Z_3 2 Z_2 2 (Z_3 3 Z_2) 2 (Z_3 2 Z_2) 2 (Z_3 3 Z_2)$

$5 (Z_3 2 Z_2) Z_3 3 Z_2 12 (Z_3 2 Z_2)$

Number of Group: 201

Order: 661308337110641113200724953626449817169452332 19772

98485248000000000

Factored Order: $2^{39}3^{57}5^{9}7^{5}11^{3}13^{3}17^{2}19^{2}23\ 29\ 31\ 37$

Composition Series: $Z_2 A_{40} Z_2 39 Z_3$

ACKNOWLEDGMENTS

The author received financial support from the New Zealand Accident Compensation Commission.

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